Hitoshi Konno*
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606, Japan.
YITP/K-1118

Spin generalization of the relativistic Calogero-Sutherland model is constructed by using the affine Hecke algebra and shown to possess the quantum affine symmetry $U_q(\widehat{\mathfrak{gl}}_2)$. The spin-less model is exactly diagonalized by means of the Macdonald symmetric polynomials. The dynamical density-density correlation function as well as one-particle Green function are evaluated exactly. We also investigate the finite-size scaling of the model and show that the low-energy behavior is described by the $C = 1$ Gaussian theory. The results indicate that the excitations obey the fractional exclusion statistics and exhibit the Tomonaga-Luttinger liquid behavior as well.

Recently, Yangian symmetry has been extensively studied [1–3] in the relation to the Calogero-Sutherland model (CSM) [4], the Haldane-Shastry model (HSM) [5] as well as conformal field theory (CFT). Especially, it is remarkable that a new structure of CFT called spinon structure has been understood based on this symmetry [3]. This motived the author in [6] to analyze the analogous structure of the level-1 integrable highest weight modules of the quantum affine algebra $U_q(\widehat{\mathfrak{sl}}_2)$. These modules are known to have a deep connection with integrable spin chain models [7]. The level-0 action of $U_q(\widehat{\mathfrak{sl}}_2)$ in the level-1 modules plays the same role as the Yangian in CFT. Namely the level-1 modules are completely reducible with respect to the level-0 action. However, no physical models related to this level-0 symmetry have been discussed.

One of the purpose of this Letter is to propose a model having this symmetry. We consider the trigonometric limit of the Ruijsnaars-Schneider model [8], which may be considered as a relativistic extension of CSM. Let θ_j $(j = 1, 2, .., N)$ be the rapidity variables and x_j be their canonically conjugate variables. We impose the canonical commutation relations $[x_j, \theta_l] = i\delta_{j,l}$ with $\hbar = 1$ and use the representation $\theta_l = -i\partial/\partial x_l$. The model is described by the following Hamiltonian H and momentum operator P

$$H = \frac{c^2}{2}(H_{-1} + H_1), \qquad P = \frac{c}{2}(H_{-1} - H_1) \tag{1}$$

with $N-$independent integrals of motion H_k (or H_{-k}) $(k = 1, 2, .., N)$

$$H_{\pm k} = \sum_{\substack{I\subset\{1,2,...,N\}\\ |I|=k}} \prod_{\substack{i\in I\\ j\notin I}} \left(\frac{\sin\frac{\alpha}{2}(x_i - x_j \mp ig/c)}{\sin\frac{\alpha}{2}(x_i - x_j)}\right)^{1/2} e^{1/c\sum_{i\in I}\theta_i} \prod_{\substack{l\in I\\ m\notin I}} \left(\frac{\sin\frac{\alpha}{2}(x_m - x_l \mp ig/c)}{\sin\frac{\alpha}{2}(x_m - x_l)}\right)^{1/2}. \tag{2}$$

Here c being the speed of light and $\alpha \in \mathbb{R}_{>0}$, $g \in \mathbb{Q}$. We normalize the mass equal to one. The model possesses the Lorentz boost generator $B = -\frac{1}{c}\sum_{i=1}^N x_i$, and is Poincaré invariant in the sense that the operators H, P, and B satisfy the Poincaré algebra:

$$[H, P] = 0, \qquad [H, B] = iP, \qquad [P, B] = i\frac{H}{c^2}. \tag{3}$$

In the non-relativistic limit $c \to \infty$, we recover the Hamiltonian of CSM as

$$\lim(H - Nc^2) = -\sum_{j=1}^N \frac{1}{2}\left(\frac{\partial}{\partial x_j}\right)^2 + \frac{g(g-1)}{4} \sum_{1\le j<k\le N} \frac{\alpha^2}{\sin^2\frac{\alpha}{2}(x_j - x_k)}$$

with identification $\alpha = 2\pi/L$, where L is the length of a ring on which particles are confined.

It is also known that the integrals of motions H_k can be gauge transformed to the Macdonald operators [9]. Let us define new parameters $p = e^{-\alpha/c}$, $t = p^g$ and new variables $z_j = e^{i\alpha x_j}$, $p^{\pm\vartheta_j} = e^{\mp\alpha z_j\partial/c\partial z_j}$. Notice the relation $p^{\pm\vartheta_j} z_j = p^{\pm 1} z_j$. Then, by using the function

$$\Delta = \prod_{\substack{j,k=1\\ j\neq k}}^N \frac{(z_j/z_k; p)_\infty}{(tz_j/z_k; p)_\infty}, \tag{4}$$

one has [10]

$$\Delta^{-1/2} H_{\pm k} \Delta^{1/2} = t^{\mp k(N-1)/2} D_k(p^{\pm 1}, t^{\pm 1}). \tag{5}$$

Here $D_k(p,t)$ are the Macdonald operators defined by [9]

$$D_k(p,t) = t^{k(k-1)/2} \sum_{\substack{I \subset \{1,2,..,N\} \\ |I|=k}} \prod_{\substack{i \in I \\ j \in\notin I}} \frac{tz_i - z_j}{z_i - z_j} \prod_{i \in I} p^{\vartheta_i}. \tag{6}$$

Now let us discuss a spin generalization of the model and clarify its quantum affine symmetry. The model we will consider is essentially the trigonometric model discussed by Bernard et.al. [2]. But we here slightly modify it. Let us consider the trigonometric solution $\overline{R}(z)$ [7] of the Yang-Baxter equation and the operator $L_{0i}(z)$ $(i = 1, 2, .., N)$ defined by

$$L_{0i}(z) = \frac{1 - q^2 z}{(1-z)q} \overline{R}_{0i}(z) = \frac{z S_{0i}^{-1} - S_{0i}}{1 - z} P_{0i}. \tag{7}$$

with $P_{ij}(v_i \otimes v_j) = v_j \otimes v_i$ and

$$S(z) = \begin{pmatrix} -q^{-1} & & & \\ & q - q^{-1} & -1 & \\ & -1 & 0 & \\ & & & -q^{-1} \end{pmatrix}. \tag{8}$$

With V_j $(j = 0, 1, .., N)$ being two dimensional vector spaces, $\bar{L}_{0i}(z)$ is regarded as a linear operator on $V_0 \otimes V_i$. Note also that the operators S_{jj+1} $(j = 1, 2, .., N-1)$ satisfy the Hecke algebra relations.

$$\begin{aligned} &S_{jj+1} - S_{jj+1}^{-1} = q - q^{-1}, \\ &S_{jj+1}\ S_{kk+1} = S_{kk+1}\ S_{jj+1} \qquad |j-k| > 1, \\ &S_{jj+1}\ S_{j+1j+2}\ S_{jj+1} = S_{j+1j+2}\ S_{jj+1}\ S_{j+1j+2}. \end{aligned} \tag{9}$$

Define the monodromy matrix $L_0(z)$ by

$$L_0(z) = L_{01}(z) L_{02}(z) \cdots L_{0N}(z). \tag{10}$$

Then the operators $\overline{R}(z)$ and $L_0(z)$ satisfy the relation

$$\overline{R}_{00'}(z/z') L_0(z) L_{0'}(z') = L_{0'}(z') L_0(z) \overline{R}_{00'}(z/z'). \tag{11}$$

We use this relation to realize the quantum affine symmetry $U_q(\widehat{\mathfrak{gl}}_2)$ as well as to define an integrable spin generalization of the model. For this purpose, we introduce the affine Hecke algebra $\hat{H}_N(q)$ [2]. The algebra $\hat{H}_N(q)$ is generated by g_{jj+1} $(j = 1, 2, .., N-1)$ and y_j $(j = 1, 2, .., N)$ with the relations (9) for g_{jj+1} and

$$\begin{aligned} &y_j\ y_k = y_k\ y_j, \\ &g_{jj+1}\ y_j\ g_{jj+1} = y_{j+1}, \\ &[g_{jj+1}, y_k] = 0, \qquad (j, j+1 \neq k). \end{aligned} \tag{12}$$

We use the following representation of $\hat{H}_N(q)$ [6].

$$\begin{aligned} g_{jk}^{\pm 1} &= \frac{q z_j - q^{-1} z_k}{z_j - z_k} (1 - K_{jk}) - q^{\mp 1} \\ y_j &= r_{jj+1}^{-1} \cdots r_{jN}^{-1}\ p^{\vartheta_j}\ r_{1j} \cdots r_{j-1j} \end{aligned}$$

with $K_{jk} f(..., z_j, .., z_k, ...) = f(..., z_k, .., z_j, ...)$ and $r_{jk} = K_{jk} g_{jk}$.

Since the operators y_j $(j = 1, \ldots, N)$ commute with each other, the 'quantized' monodromy matrix [2]

$$\widehat{L}_0(z) = L_{01}(z y_1) \cdots L_{0N}(z y_N) \tag{13}$$

also satisfies the relation (11). Consider the formal expansion of $\widehat{L}_0(z)$ in $z^{\pm 1}$ and define

$$\widehat{L}_0^{\pm}(z) = \sum_{\pm n \geq 0} z^n \begin{pmatrix} l_{11}^{\pm}[n] & l_{12}^{\pm}[n] \\ l_{21}^{\pm}[n] & l_{22}^{\pm}[n] \end{pmatrix}. \tag{14}$$

From (7) and (11), we have the relations $l_{21}^{+}[0] = l_{12}^{-}[0] = 0$ and $l_{jj}^{+}[0] l_{jj}^{-}[0] = 1 \quad (j = 1, 2)$ as well as

$$\overline{R}_{00'}(z/z')\widehat{L}_0^{\pm}(z)\widehat{L}_{0'}^{\pm}(z') = \widehat{L}_{0'}^{\pm}(z')\widehat{L}_0^{\pm}(z)\overline{R}_{00'}(z/z'), \tag{15}$$

$$\overline{R}_{00'}(z/z')\widehat{L}_0^{+}(z)\widehat{L}_{0'}^{-}(z') = \widehat{L}_{0'}^{-}(z')\widehat{L}_0^{+}(z)\overline{R}_{00'}(z/z'). \tag{16}$$

Now let $\mathcal{F}_N$ be the space of vectors $v \in \{f(z_1, z_2, .., z_N) \otimes V^{\otimes N}\}$ satisfying

$$(g_{j j+1} - S_{j j+1})v = 0 \qquad j = 1, 2, .., N-1. \tag{17}$$

The relations (15) and (16) define a level-0 action $U_q(\widehat{\mathfrak{gl}}_2)_0$ on $\mathcal{F}_N$ [6]. From (13) and (14), we obtain the following realization:

$$\begin{aligned}
\pi^{(N)}(e_0) &= \sum_{j=1}^{N} y_j^{-1} q^{h_1} \otimes \cdots \otimes q^{h_1} \otimes \overset{j}{\check{f_1}} \otimes q^{h_2} \otimes \cdots \otimes q^{h_2}, \\
\pi^{(N)}(f_0) &= \sum_{j=1}^{N} y_j q^{-h_2} \otimes \cdots \otimes q^{-h_2} \otimes \overset{j}{\check{e_1}} \otimes q^{-h_1} \otimes \cdots \otimes q^{-h_1}, \\
\pi^{(N)}(e_1) &= \sum_{j=1}^{N} q^{h_2} \otimes \cdots \otimes q^{h_2} \otimes \overset{j}{\check{e_1}} \otimes q^{h_1} \otimes \cdots \otimes q^{h_1}, \\
\pi^{(N)}(f_1) &= \sum_{j=1}^{N} q^{-h_1} \otimes \cdots \otimes q^{-h_1} \otimes \overset{j}{\check{f_1}} \otimes q^{-h_2} \otimes \cdots \otimes q^{-h_2}, \\
\pi^{(N)}(q^{\pm h_j}) &= q^{\pm h_j} \otimes \cdots \otimes q^{\pm h_j} \qquad j = 1, 2,
\end{aligned}$$

where $e_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $f_1 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $h_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $h_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

The center of $U_q(\widehat{\mathfrak{gl}}_2)_0$ is given by the quantum determinant q-det$\widehat{L}_0(z)$. Direct calculation shows

$$q\text{-det}\widehat{L}_0(z) = q^N \prod_{j=1}^{N} \frac{(1 - q^{-1} y_j z^{-1})}{(1 - q y_j z^{-1})}. \tag{18}$$

Expanding q-det$\widehat{L}_0(z)$ in the power of z^{-1}, one gets the commuting family of $N-$independent operators

$$\sum_{i_1 < \cdots < i_k} y_{i_1} \cdots y_{i_k} \quad (k = 1, 2, .., N). \tag{19}$$

Now we define a model on $\mathcal{F}_N$ by the following Hamiltonian $\hat{h}$ and momentum operator $\hat{p}$

$$\hat{h} = \frac{c^2}{2} \sum_{j=1}^{N} (y_j^{-1} + y_j), \quad \hat{p} = \frac{c}{2} \sum_{j=1}^{N} (y_j^{-1} - y_j). \tag{20}$$

Defining also the operator $\hat{b} = \frac{i}{c \ln p} \sum_{j=1}^{N} \ln z_j$, one can easily show that $\hat{h}$, $\hat{p}$ and $\hat{b}$ satisfy the Poincaré algebra (3). Furthermore, in the spin-less sector of $\mathcal{F}_N$, for example $\{f_{sym}(z_1, .., z_N) \otimes v_+ \otimes \cdots \otimes v_+\}$ with f_{sym} being symmetric functions, $\hat{h}$, $\hat{p}$ as well as all the integrals of motion (19) of the model coincide with those of the relativistic Calogero-Sutherland model (1)~(2). This is due to the following formula [6].

$$D_k(p^{\pm 1}, t^{\pm 1}) = (-t^{1/2})^{\pm k(N-1)} \sum_{i_1 < \cdots < i_k} y_{i_1}^{\pm 1} \cdots y_{i_k}^{\pm 1},$$

where we made identification $t=q^2$. From (5), this implies $H=\Delta^{1/2}\,\hat{h}\,\Delta^{-1/2}$ and $P=\Delta^{1/2}\,\hat{p}\,\Delta^{-1/2}$. We hence have obtained the integrable spin generalization of the relativistic Calogero-Sutherland model and shown that it possesses the quantum affine symmetry $U_q(\widehat{\mathfrak{gl}_2})_0$.

We next consider the diagonalization of the spin-less model and evaluate the dynamical correlation functions. The diagonalization of the integrals of motion (2) can be carried out by the Macdonald symmetric polynomials. Let $\lambda=(\lambda_1,\cdots,\lambda_N)$, $\lambda_1\geq\cdots\geq\lambda_N\geq 0$ be a partition and denote the Macdonald symmetric polynomial by $P_\lambda(z;p,t)$. Then one has [9],

$$D_k(p^{\pm1},t^{\pm1})P_\lambda(z;p,t)=\Big(\sum_{i_1<\cdots<i_k}\prod_{l=1}^{k}t^{N-i_l}p^{\lambda_l}\Big)P_\lambda(z;p,t).$$

Therefore, from (5), we obtain the exact eigen values of H and P as

$$E_N(\lambda)=c^2\sum_{j=1}^{N}\mathrm{ch}\theta_j,\quad P_N(\lambda)=c\sum_{j=1}^{N}\mathrm{sh}\theta_j \tag{21}$$

$$\theta_j=\frac{2\pi}{Lc}\Big\{\lambda_j+g\Big(\frac{N+1}{2}-j\Big)\Big\}, \tag{22}$$

where we set $\alpha=2\pi/L$. The corresponding eigen functions are given by

$$\Psi_\lambda(z)=\Delta^{1/2}P_\lambda(z;p,t). \tag{23}$$

The model thus can be regarded as an ideal gas of $N-$relativistic pseudo-particles with the pseudo-rapidities (22). One should note that the formula (22) obey the following Bethe ansatz like equations

$$Lc\theta_j=2\pi I_j+\pi(g-1)\sum_{l=1}^{N}\mathrm{sgn}(\theta_j-\theta_l), \tag{24}$$

with $I_j=\lambda_j+\frac{N+1}{2}-j$.

The ground state is given by the function $\Psi_\phi(z)=\Delta^{1/2}$ corresponding to the empty partition $\lambda=\phi$. The ground state momentum and energy eigenvalues are evaluated as $P_N^{(0)}=0$ and

$$E_N^{(0)}=c^2\mathrm{sh}\,\frac{\pi gN}{cL}/\mathrm{sh}\,\frac{\pi g}{cL}. \tag{25}$$

Hence the ground state can be described as a filled Fermi sea with pseudo-momenta $P_j^{(0)}=\mathrm{sh}\theta_j$ with $-\theta_F\leq\theta_j\leq\theta_F$ $(j=1,2,..,N)$, where $\theta_F=\pi g(N-1)/Lc$.

The dynamical density-density correlation functions as well as one-particle Green function can be evaluated by making use of the Macdonald symmetric polynomial. We here summarize the results. To each partition λ, we assign a Young diagram $\mathcal{D}(\lambda)=\{(i,j)|1\leq i\leq l(\lambda),\ 1\leq j\leq\lambda_i,\quad i,j\in\mathbb{Z}_{>0}\}$. Let λ' be the conjugate partition of λ. For each cell $\gamma=(i,j)$ of $\mathcal{D}(\lambda)$, we define the quantities $a(\gamma)=\lambda_i-j$, $a'(\gamma)=j-1$, $l(\gamma)=\lambda'_j-i$ and $l'(\gamma)=i-1$. Then we have

$$\langle 0|\rho(\xi,t)\rho(0,0)|0\rangle=\frac{A_N}{L^2}\sum_\lambda\frac{(1-p^{|\lambda|})^2(\chi^\lambda(p,t))^2}{h_\lambda(p,t)h_{\lambda'}(t,p)}\mathcal{N}(\lambda)\cos(\mathcal{P}(\lambda)\xi)e^{-i\mathcal{E}(\lambda)t}, \tag{26}$$

$$\langle 0|\Psi^\dagger(\xi,t)\Psi(0,0)|0\rangle=\frac{A_{N+1}}{A_N}\sum_\lambda\frac{t^{2|\lambda|}\Big((t^{-1})_\lambda^{(p,t)}\Big)^2}{h_\lambda(p,t)h_{\lambda'}(t,p)}\mathcal{N}(\lambda)e^{-i(\mathcal{E}(\lambda)t-\mathcal{P}(\lambda)\xi)}, \tag{27}$$

with ξ being a real coordinate conjugate to P, $|\lambda|=\sum\lambda_j$, $\mathcal{E}(\lambda)=E_N(\lambda)-E_N^{(0)}$, $\mathcal{P}(\lambda)=P_N(\lambda)$ and

$$A_N=\prod_{j=1}^{N}\frac{(pt^{j-1};p)_\infty(t;p)_\infty}{(t^j;p)_\infty(p;p)_\infty},$$
$$h_\lambda(p,t)=\prod_{\gamma\in\lambda}\Big(1-p^{a(\gamma)}t^{l(\gamma)+1}\Big),\qquad h_{\lambda'}(t,p)=\prod_{\gamma\in\lambda}\Big(1-p^{a(\gamma)+1}t^{l(\gamma)}\Big),$$

$$\mathcal{N}(\lambda) = \prod_{\gamma \in \lambda} \frac{1 - p^{a'(\gamma)} t^{N - l'(\gamma)}}{1 - p^{a'(\gamma)+1} t^{N - l'(\gamma) - 1}},$$

$$\chi^\lambda(p,t) = \prod_{\substack{\gamma \in \lambda \\ \gamma \neq (1,1)}} \left(t^{l'(\gamma)} - p^{a'(\gamma)} \right), \qquad (a)_\lambda^{(p,t)} = \prod_{\gamma \in \lambda} \left(t^{l'(\gamma)} - p^{a'(\gamma)} a \right).$$

For the rational coupling $g = r/s$, one should remark that the factor $\chi^\lambda(p,t)$ (resp.$(t^{-1})_\lambda^{(p,t)}$) vanishes if the diagram $\mathcal{D}(\lambda)$ contains the lattice point $(s+1, r+1)$(resp.$(s, r+1)$). According to the same argument as by Ha in CSM [11], this indicates that in the thermodynamic limit, only the states which contains minimal r quasi-hole excitations accompanied by s(resp. $s-1$) quasi-particles can contribute as the intermediate states in (26) (resp.(27)). One can thus conclude that the excitations of the model obey the fractional exclusion statistics á la Haldane [12] as in CSM [11].

Furthermore, the exact spectrums (21) allows one to analyze the finite-size scaling of the model in the thermodynamic limit, N, $L \to \infty$ with $N/L = n$ fixed. First of all, from (25) we obtain the finite-size correction to the ground state energy as

$$\lim E_N^0 = L\varepsilon_0 - \frac{\pi v}{6L} g + O(\frac{1}{L^2}), \tag{28}$$

where $\varepsilon_0 = \frac{c^3}{\pi g} \mathrm{sh} \frac{\pi g n}{c}$ and $v = c\ \mathrm{sh} \frac{\pi g n}{c}$ are the ground state energy density and the velocity of the elementary excitation, respectively. In comparison with the general theory [13], one may suspect that the central charge is given by g. However this is not the correct identification [14]. The central charge should be identified with 1. This can be justified by calculating the low temperature expansion of the free energy from (24). Instead, we here justify it by deriving the whole conformal dimensions associated with the elementary excitations. These can be obtained by evaluating the differences of the total energy and momentum from the groung state eigenvalues under the change of the particle number (by ΔN) and the transfer of the $\Delta D-$particles from the left Fermi point to the right one [14]. We hence obtain the finite-size corrections

$$\Delta E = \mu \Delta N + \frac{2\pi v}{L} \Big[\frac{g}{4} \Delta N^2 + \frac{1}{g} \Big(\Delta D + \frac{\Phi}{2\pi} \Big)^2 \Big],$$

$$\Delta P = 2 p_F \Delta D + \frac{2\pi \mathrm{ch} \frac{\pi g n}{c}}{L} \Delta N \Big(\Delta D + \frac{\Phi}{2\pi} \Big),$$

where $\mu = c^2 \mathrm{ch} \frac{\pi g n}{c}$ and $p_F = v/g$ are the chemical potential and the Fermi momentum, respectively. We here modified the argument by Kawakami and Yang by considering the flux excitations Φ associated with the change of the particle number ΔN [15,11]. Adding the contribution from the quasi-particle (N^+) and quasi-hole(N^-) excitations, we finally obtain the right and left conformal dimensions $h^\pm$ as follows.

$$h^\pm(\Delta N; \Delta D; N^\pm) = \frac{1}{2} \Big[\frac{\sqrt{g} \Delta N}{2} \pm \frac{1}{\sqrt{g}} \Big(\Delta D + \frac{\Phi}{2\pi} \Big) \Big]^2 + N^\pm. \tag{29}$$

Remarkablly, the result does not depend on c. Note that the flux carried by a particle is πg as in CSM [15] so that $\Phi = \pi g \Delta N$. One can thus write (29) as

$$h^+ = \frac{1}{2g} (\Delta D + g \Delta N)^2 + N^+, \ h^- = \frac{1}{2g} \Delta D^2 + N^-.$$

This result indicates that the effect of the flux excitation is equivalent to impose the new selection rule $\Delta D = \frac{g}{2} \Delta N$ (mod 1) on (29) without $\Phi/2\pi$. Notice that this selection rule can be obtained from the periodicity of the plane wave $\exp(i\theta_j x_j)$ under the change $x_j \to x_j + Lc$. Hence $h^\pm$ with $N^\pm = 0$ can be regarded as the conformal dimensions of the $U(1)-$primary fields in the $C = 1$ Gaussian theory. From the results (26) and (27), we also have succeeded to obtain the thermodynamic limit of the dynamical correlation functions and their asymptotic form. The critical exponents thus obtained agree with Ha's results [11] as well as those obtained from $h^\pm$ with assignment $\Delta N = 0$ for the density correlation and $\Delta N = 1$ for one-particle Green function. One can thus conclude that the model possesses the Tomonaga-Luttinger liquid property [16].

In the case with the special coupling $g = 2$, the Gaussian theory is known to become the level-1 $su(2)$ Wess-Zumino-Witten theory. This feature is consistent with the results in Ref. [6], where the setting $t = p^2$, is inevitable to define a new level-0 action of $U_q(\widehat{\mathfrak{sl}}_2)$.

In comparison with CSM, our model possesses one extra paprmeter c. The ultra relativistic limit $c \to 0$ is especially interesting. There one has a decoupling of the left and right-movers. In addition, the limit $g \to 0$ with g/c fixed reduces the Macdonald polynomial to the Hall-Littlewood function [9]. This suggests that a certain mathematical structre remains in this limit [17]. The detailed investigation will be discussed elsewhere.

The author would like to thank P.J. Forrester, T. Fukui, M. Jimbo, N. Kawakami, V.B. Kuznetsov, T. Miwa, K. Ueno and T. Yamamoto for valuable discussions.

* Yukawa Fellow.

---

[1] F.D.M. Haldane, Z.N.C. Ha, J.C. Talstra, D. Bernard and V. Pasquier, Phys. Rev. **69**, 2021, (1992).
[2] D. Bernard, M. Gaudin, F. D. M. Haldane and V. Pasquier, J. Phys. A, **26**, 5219, (1993).
[3] D. Bernard, V. Pasquier and D. Serban, Nucl. Phys.,**B428**, 612, (1994); P. Bouwknegt, A.W.W. Ludwig and K. Schoutens, Phys. Lett. **338B**, 448, (1994).
[4] F. Calogero, Jour.Math.Phys., **12**, 419, (1971); B. Sutherland, Phys.Rev., **A5**,1372,(1972).
[5] F.D.M. Haldane, Phys.Rev.Lett.,**60**, 635, (1988); B.S. Shastry, Pys.Rev.Lett., **60**, 639, (1988).
[6] M. Jimbo, R. Kedem, H. Konno, T. Miwa and J.-U.H. Petersen, Preprint, RIMS-1016, q-alg/9505009, 1995; RIMS-1019, YITP/K-1111, q-alg/9506016, 1995, to be published in J.Phys.A.
[7] M. Jimbo and T. Miwa, *Algebraic Analysis of Solvable Lattice Models*, CBMS Regional Conference Series in Mathematics vol. 85, AMS, 1994.
[8] S.N.M. Ruijsenaars and H. Schneider, Ann.Phys., **170**, 370, (1986); S.N.M. Ruijsenaars, Comm.Math.Phys., **110**, 191, (1987).
[9] I.G. Macdonald, *Symmetric functions and Hall polynomials, 2nd ed.*, Clarendon Press, 1995.
[10] J.F. van Diejen, Preprint, q-alg/9504012, 1995.
[11] Z.N.C. Ha, Phys.Rev.Lett., **73**, 1574, (1994); Nucl.Phys.**B435**[FS], 604, (1995).
[12] F.D.M. Haldane, Phys.Rev.Lett., **67**, 937, (1991).
[13] H.W. Blöte, J.L. Cardy and M.P. Nightingale, Phys.Rev.Lett., **56**, 724, (1986); I. Affleck, Phys.Rev.Lett., **56**, 746, (1986).
[14] N. Kawakami and S.-K. Yang, Phys. Rev. Lett, **67**, 2493, (1991).
[15] F.D.M. Haldane, *in* Proc. 16th Taniguchi Symp., Kashikojima, Japan, October 26-29, 1993, ed. A.Okiji and N.Kawakami (Springer, Berlin, 1994).
[16] F.D.M. Haldane, Phys.Rev.Lett, **47**, 1840, (1981).
[17] J.Shiraishi, H.Kubo, H.Awata and S.Odake, Preprint, YITP/U-95-30, q-alg/9507034,1995.